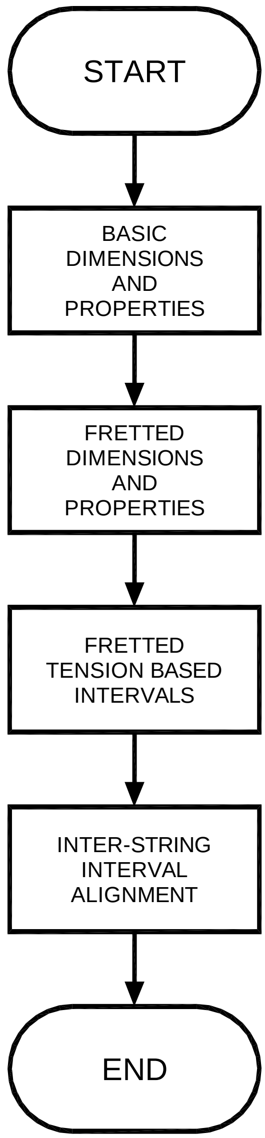
Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

by Gary Magliari



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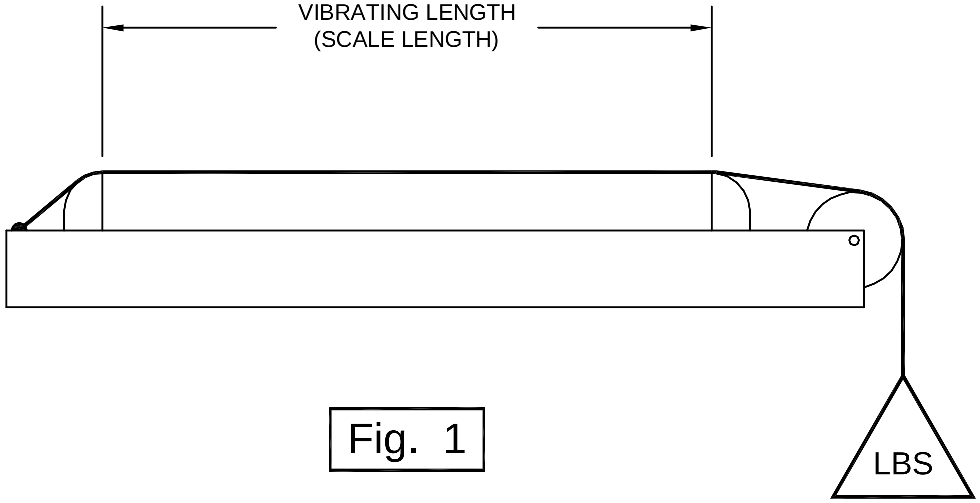

Fig. 1

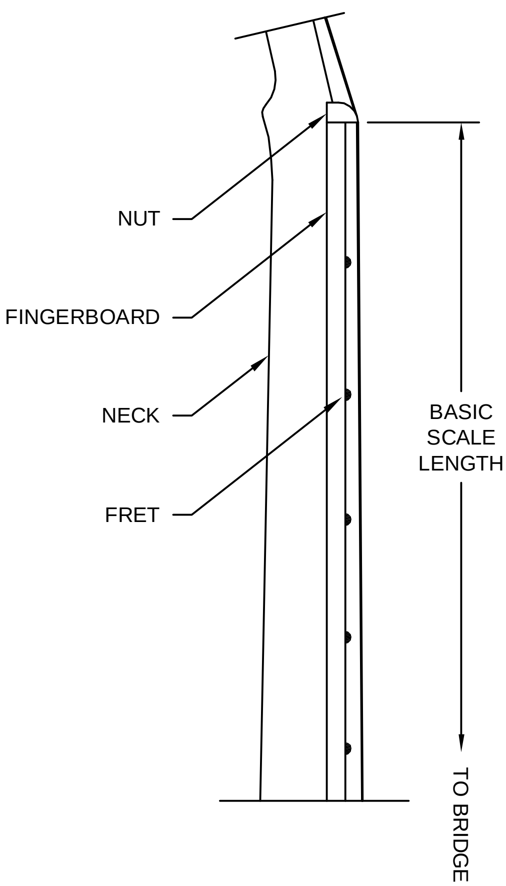

Fig. 2a

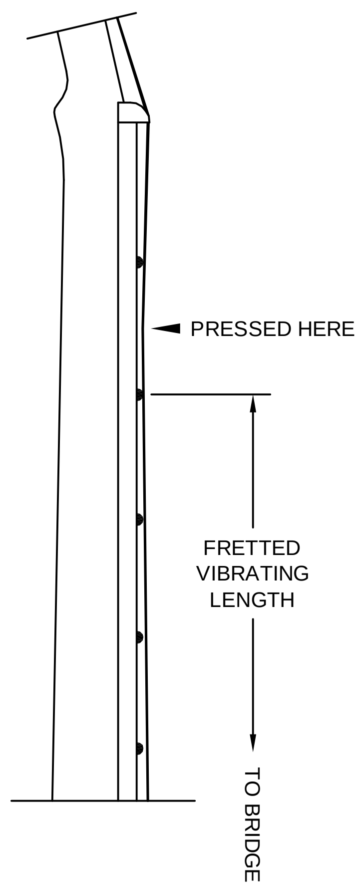

Fig. 2b

Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

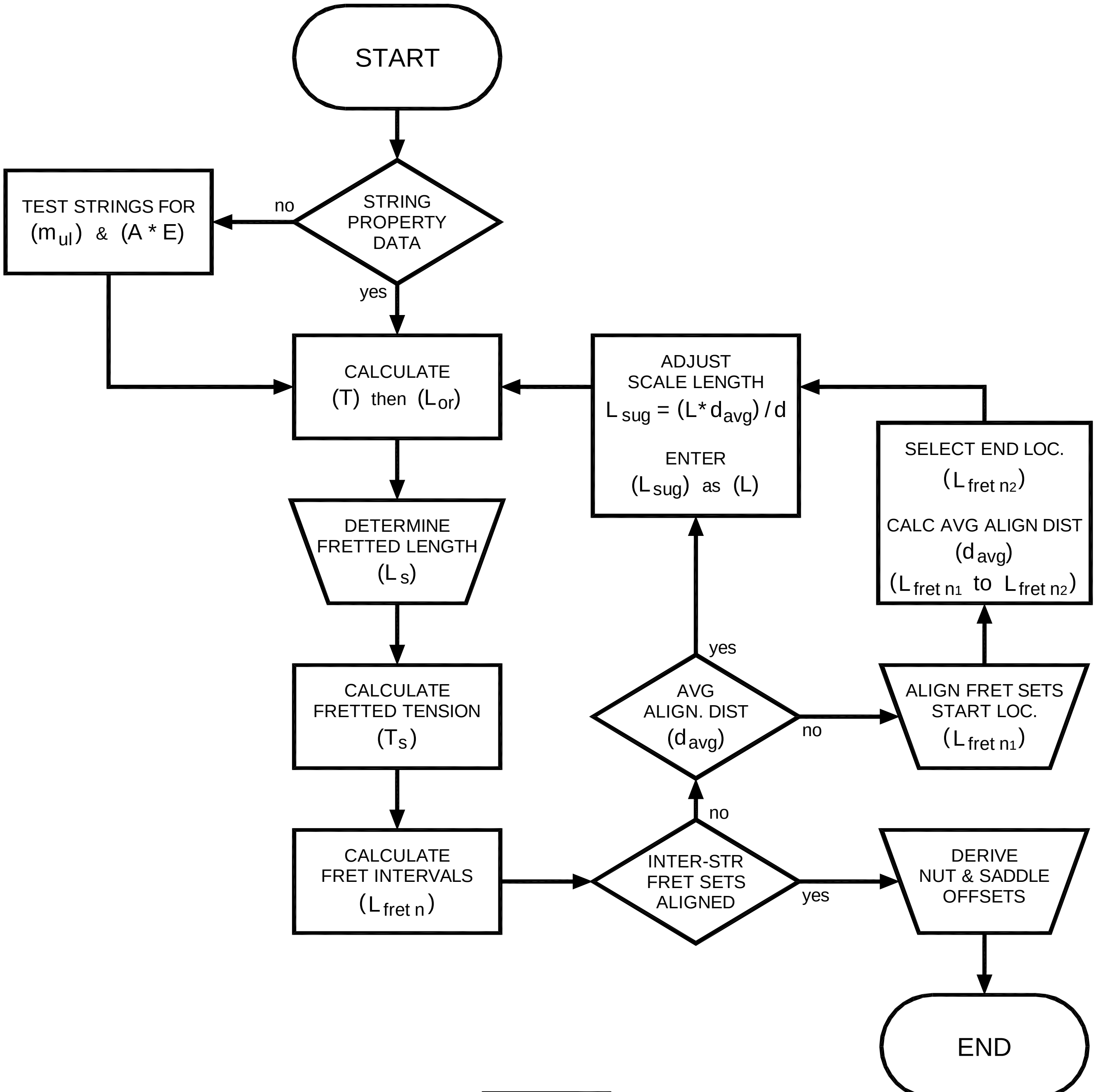

Fig. 3

Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

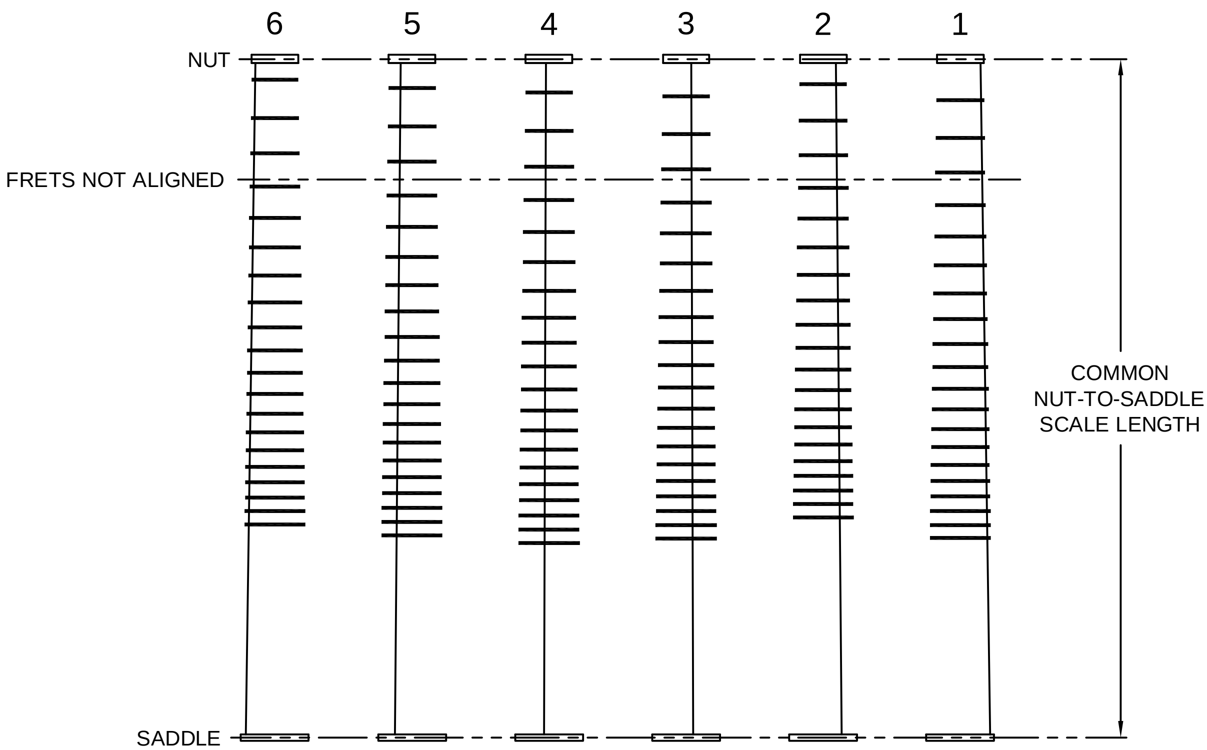

Fig. 4

Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

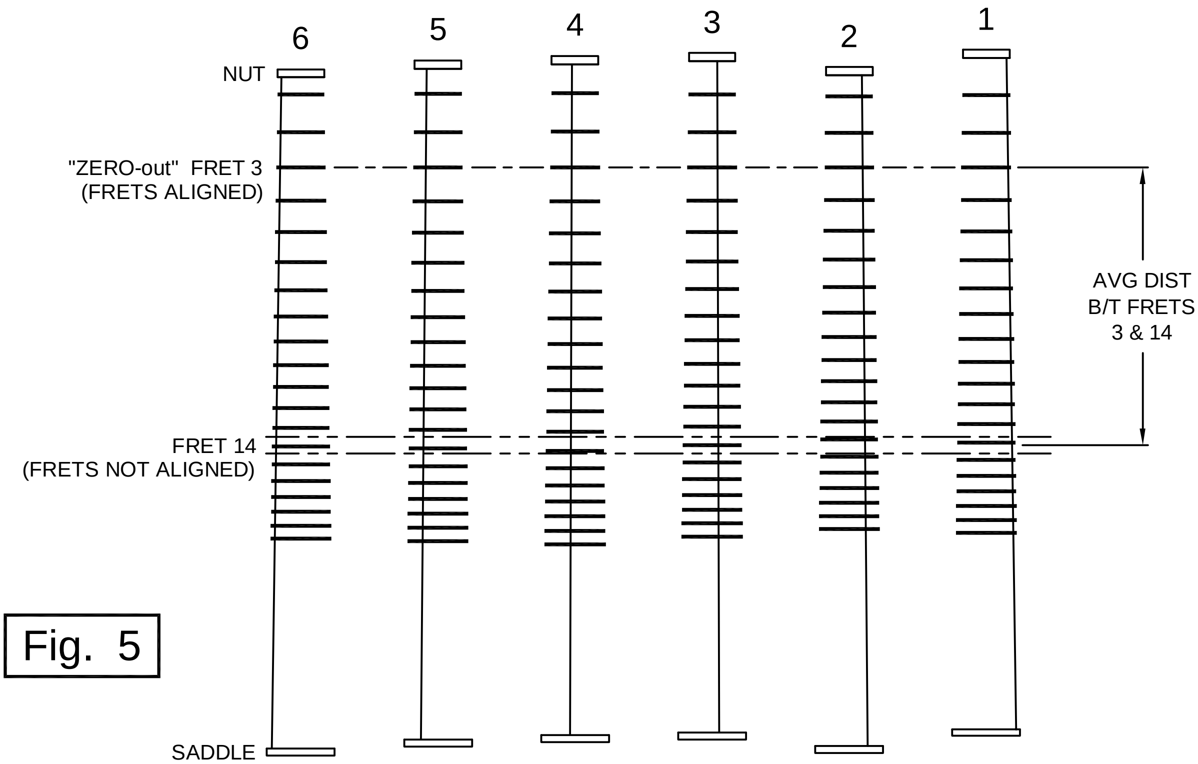

Fig. 5

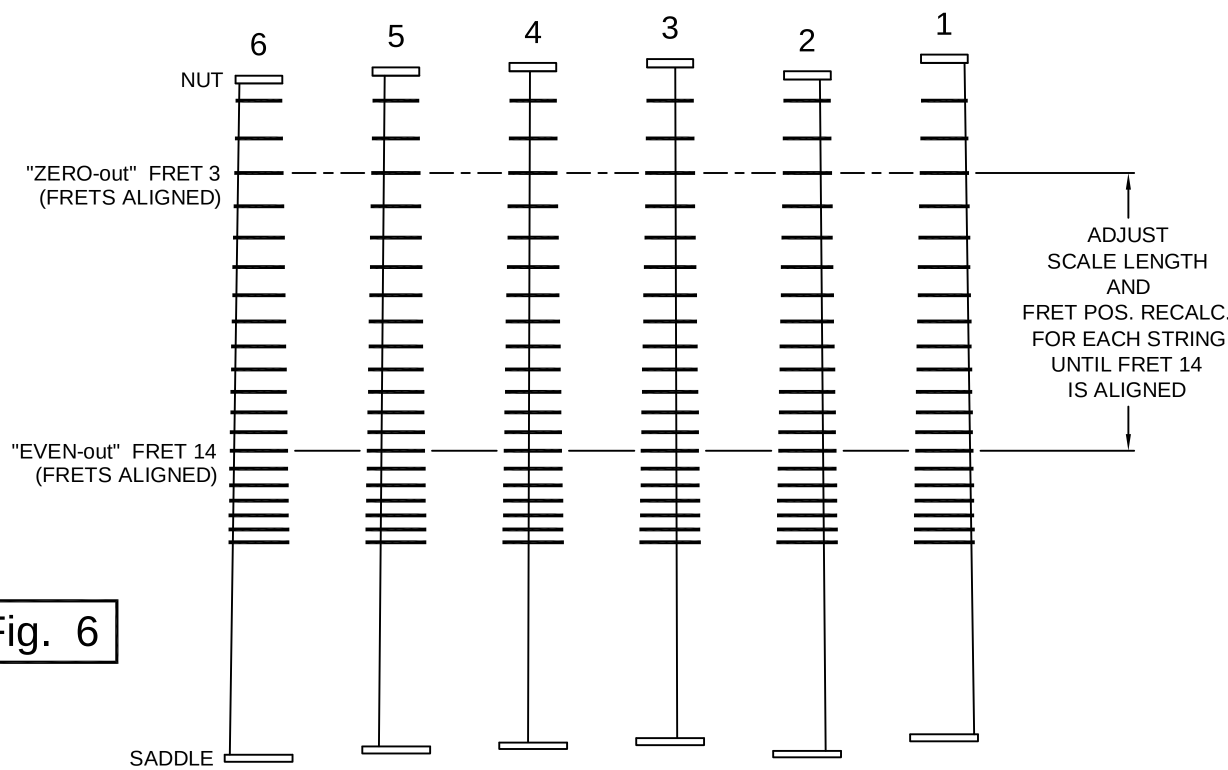

Fig. 6

Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

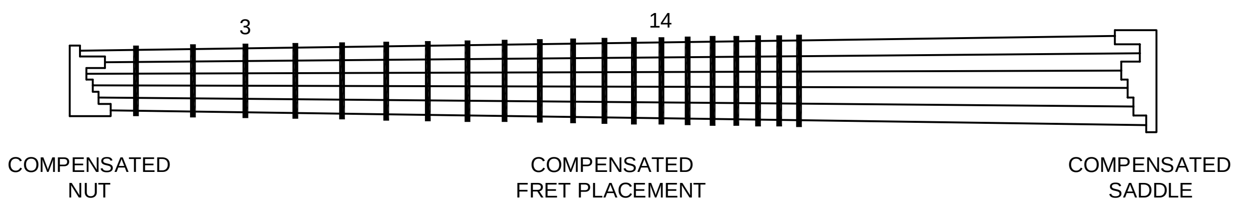

Fig. 7

Method for Positioning Musical Instrument Frets That Compensate for Fretting-Induced String Tension

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Overview

The procedure disclosed in this paper specifically applies to stringed musical instruments with fretted fingerboards. Though the process relates to all fretted instruments, I will be discussing guitars in particular to explain the concept. All formulae shown are not depicted in typical mathematical notation but are written in a manner that would make them easier to enter directly into a computer spreadsheet application such as Excel, etc. The user need only substitute actual numerical values or *"cell addresses"* in place of formulae variables.

Background of the Procedure

When a musician picks up a guitar and places their hand on the fingerboard, then presses a string down to a fret and plucks that string, it creates a sound with a pitch that is specific to that string and fret position. The fret positions or intervals have traditionally been determined by a mathematical relationship known as the *"rule of 18"*, or more precisely: 17.817153. The rule basically states that:

> *If you divide the scale length (the distance from the nut to the bridge) by 17.817, you end up with the first fret position starting from the nut end of the fingerboard. If you then take the scale length minus the first fret distance and divide the remainder by 17.817, you end up with the second fret position located from the first fret. Next if you take the scale length minus the first and second fret distances and divide the*

remainder by 17.817, you end up with the third fret position located from the second fret. This process is repeated until all frets required are located.

Mathematically this is known as a *"geometric progression"*. It is based on the 12^{th} root of 2 or *"2^(1/12)"*, which equals 1.059463. The *"12"* comes from the twelve steps or intervals between octaves; A, A#, B, C, C#, D, D#, E, F, F#, G, G# & A. The number 1.059463 is the ratio between fret intervals. For example, on a guitar with a 25.5" scale length, the distance between the nut and the first fret is 1.4312. The distance between the first and second frets is 1.3509. If you divide the two *(1.4312/1.3509)* you get 1.0594. This value is fundamental to a system of tuning known as ***"twelve-tone equal temperament"*** *(12-TET)* and is the basis for all western music. It affects all musical instruments designed for the *equal tempered scale*.

Another way to calculate fret position or vibrating length as measured from the bridge this time would be as follows:

$$L_0 = \text{Scale Length} = 25.500$$
$$L_1 = L_0 / 1.059463 = 24.069$$
$$L_2 = L_1 / 1.059463 = 22.718$$
$$L_3 = \text{etc…}$$

where,

$L_1 = 1^{st}$ *fret*

$L_2 = 2^{nd}$ *fret*

$L_3 = 3^{rd}$ *fret, etc…*

A variation of the formula above can be used to calculate the frequencies *(Hz)* of the notes in the *equal tempered scale*. Starting with a particular note such as "A_4" *(440 Hz.)* we can find the following:

If,

$$A_4 = 440 \text{ Hz.}$$

then,

$$A^{\#}_4 = 440 * 1.059463$$

$$A^{\#}_4 = 466.16 \text{ Hz.}$$

When the mathematics described above was created for deriving pitch intervals and, as applied to stringed instruments, establishing *vibrating length* and ultimately fret locations on a fingerboard, it was based on a string with equal tension. This means that if we start with a 25.5" vibrating length *(or scale length)* and tighten the 0.013 dia. high *"E"* string until it reaches a tension force of 27.4 lbs., and we pluck the string, it will ring out at a frequency of 329.63 Hz. or a pitch of "E_4" *(see fig. 1)*. If we physically shorten the vibrating length by one fret interval *(-1.431)* and maintain the tension at 27.4 lbs. the new vibrating length will be:

$$\text{vibrating length} = 25.500 - 1.431 = 24.069$$

Knowing that the starting frequency is 329.63 Hz. (E_4) for a vibrating length of 25.5" we can easily calculate the frequency for the length shortened by one interval, i.e., 24.069", using the formula from the previous section which gives us a new higher frequency of 349.2 Hz. (F_4). But let's assume for the moment that we have no idea what the starting frequency or original vibrating length is. Assume that all we know is vibrating length *(L)*, tension *(T)*, and the string property (m_{ul}) known as *"mass per unit length" (lbs./in.)*. Using the following formula we can calculate the new frequency *(f)*:

$$f = SQRT((g * T) / m_{ul}) / (2 * L)$$

$$f = SQRT((386.089 * 27.4) / 0.00003744) / (2 * 24.069)$$

$$f = 349.2 \text{ Hz}$$

Plucking the string again will cause it to ring out at a frequency of 349.2 Hz. or a pitch of "F_4". By the way, *(g)* is the constant for gravity *(386.089 in./sec^2)* and is necessary to use in this formula if we want units of measurement to be in *"inches and pounds".*

The reason for using this alternate and slightly more complicated formula to solve for frequency *(f)* is because later on when tension compensated fret placement is discussed in more detail a variation of this formula will be used to locate frets. However what these examples thus far demonstrate is how strings of varying lengths but kept under equal tension can produce different notes. This is also what the mathematics of equal temperament are based on as applied to stringed instruments; ***"equal tension and vibrating length"***. The examples also clearly illustrate how vibrating length *(L)*, tension *(T)*, frequency *(f),* and the basic string property *(m_{ul})* relate to each other. If you know the value of any three of these variables you can easily solve for the fourth.

This is all well and good for establishing the theory and setting up the basis for calculating fret intervals, but in the real world of fretted instruments the vibrating length is controlled by pressing a string down to a particular fret. **Deflecting a string out of its *"straight-line"* nut-to-bridge equilibrium causes it to stretch** *(see fig. 2a & 2b)*. When a string is stretched its tension increases which in turn causes the pitch to increase. And now the fret, whose position was based on the string's pre-stretched tension, is not in the correct location because its actual fretted tension is higher. And therein lies the dilemma. **Fret intervals based on the *"rule of 18"* are accurate as long as you don't press a string down to a fret.**

The fret location should be recalculated based on the new higher tension. Theoretically this means shifting the fret slightly away from the bridge to increase its vibrating length thereby reducing its pitch to the correct frequency. However, traditionally the vibrating length is increased by moving the bridge or saddle away from the fret *(or nut)*. This is known typically as *"bridge compensation"* and is sometimes accomplished with the aid of an adjustable saddle. The technique involves adjusting the saddle until the note fretted and plucked at the 12th fret equals the pitch of the 1st overtone or harmonic plucked at approximately the same location, i.e., center of the vibrating length of the open string.

Traditionally this method has allowed luthiers to intonate their instruments reasonably well especially given the fact that it can be performed with little or no special equipment, just a discerning ear. It may accurately compensate for fretted tension at the 12^{th} fret, but it does not accurately compensate for fretted tension at the remaining frets. The reason for this is because fretted tension, or the amount that tension increases above the open string tension, varies from fret-to-fret. So bridge compensation corrects only by using a middle of the road location, e.g., the 12^{th} fret, and compensating for what is more or less a rough average of the fretted tension of all fret locations at once. This leaves every remaining fret location below and above the 12^{th} fret either over-compensated or under-compensated by design.

In order of importance, the two primary factors that affect the amount of compensation required are string *"stiffness" (spring rate "k")* and *"action"* or string height. The stiffness of a string is the relationship between the elastic modulus of the core material and the physical size of the core itself (cross-section and unstressed length). The stiffness determines how much the tension goes up for a given amount of stretch. This is to say that strings with greater stiffness will have a higher increase *(Δ)* in tension than strings with lower stiffness given the same amount of stretch. So consequently strings with higher stiffness and ultimately higher fretted tension require more compensation.

Action, on the other hand, determines how much the strings will stretch during fretting. A note will not ring out clearly unless the string is pressed down to a fret and slightly beyond. The action or string height above the frets varies from being low at the first fret and gradually gets higher as you go up the fingerboard. The action can also vary from string-to-string, being lower for the thinner strings and higher for the thicker ones. And the higher it is the more the strings stretch. As with stiffness, action contributes greatly to fretted tension.

What the previous few paragraphs illustrate is that fretted tension is not linear but varies depending on what string your pressing down on and at what fret location your pressing it down to. They are all different.

Investigating the intonation issues associated with fretted instruments has been an extremely enlightening experience. I can understand why so few have taken up the challenge of an in-depth study. Prior to desktop computers becoming commonplace, performing this exercise by hand would have been a long slow laborious process. Changing one or two variables could require many hours or even days of recalculation, whereas, with a computer answers come instantly.

In many cases we are dealing with very small numbers and even smaller results. An accurate and significant conclusion could be hard to come by due to rounding-off errors which tend to accumulate. For example, a medium set of strings set at a low-to-medium action will only stretch roughly between 0.0008" and 0.0035" depending on where they are fretted. These are very small numbers. Still they have a significant effect on pitch especially considering that instruments are traditionally intonated at the 12th fret. In this scenario the average stretch at the 12th fret is approximately 0.0015". What happens to the difference? It is certainly easy to overlook.

Over the years luthiers have followed time-honored methods for placing frets and intonating their instruments. They have been able to produce meaningful results that provided acceptable intonation just by compensating at the bridge or maybe tweaking the nut through trial and error. They may have even limited themselves to improving only those areas which were audibly most noticeable such as regions involving the first few strings or frets. Everyone has their methods. So why not stay with tradition?

As a player, I can tell you that there is nothing more inspiring then when everything is clicking. I mean when the feel and action are dead on, you strum a chord and it speaks back with a thunderous response and sustaining notes ring out like chimes with perfect intonation. There is nothing more satisfying. This is what players want. And I believe a very big part of that equation is intonation, without a doubt. And it is an attribute that most players are not aware of how good it can be. They usually just get moments of good intonation or areas of good intonation. They rarely, if ever, get consistently great intonation top to bottom. For this reason, and this reason alone, departing from tradition is certainly worth considering.

The effect that intonation has, whether it is good or bad, on the perception of an instrument is much greater than the sum of its parts. Its significance cannot be overstated. Take four instruments made exactly the same way, made from the same tree in fact. Put *"rule of eighteen"* fret placement on three and tension compensated fret placement on one. Allow a group of respected musicians to thoroughly evaluate them and they will pick out the latter almost every time as being the one they favor. Their reasons may vary but they all will basically equate to the same thing, *"There's just something special about this one."* **Perception!**

From a builder's perspective there is another and more practical reason to try tension compensated fret placement. Besides the fact that it will make the instrument sound better, it does not add to the cost of construction. Not in time or materials. You still have a nut, the same amount of frets, a bridge and a saddle. There are no extra parts, spring-loaded devices or contraptions to install. All you will be doing is placing the frets in a slightly different location and possibly changing the shape of the nut and saddle. The physical differences are so visually subtle that unless pointed out most players will not even notice them.

Lets summarize what has been discussed so far relative to traditional fret placement and intonation:

- The 12^{th} root of 2 or *"1.059463"* is the ratio between pitch frequencies or intervals in the *"twelve-tone equal tempered scale"*. This value also determines the spacing between frets in traditional placement.

- It is not possible to accurately intonate a fretted instrument throughout the fingerboard using *"rule of 18"* fret placement and bridge saddle compensation alone. Fretted tension is not linear but varies at every fret interval.

- There is a close relationship between vibrating length *(L)*, tension *(T)*, frequency *(f),* and the string property *(m_{ul})*. If you know the value of any three of these variables you can solve for the fourth.

- The two most important factors that determine the amount of compensation an instrument will require are string *"stiffness" (spring rate "k")* and *"action"* or string height. Greater stiffness requires more compensation for a given amount of stretch. Action determines how much the strings will stretch when they are pressed down to a fret.

- Intonation quality can have a significant effect on a player's overall perception of an instrument. Instruments with better than average intonation are *"perceived"* as being physically or mechanically superior as a whole.

Traditionally fretted tension is, and has been, compensated for at the 12th fret. By design this leaves quite a bit of error built into the fingerboard. The only simple way to correct for this inaccuracy without adding extra parts or effort is through fret compensation. Fret placement based on real string tension is a natural progression in the evolution of fretted instruments. This could virtually eliminate the intonation short comings that have been inherent to the history of fretted instruments.

<u>Tension Compensated Fret Placement</u>

The following pages will describe a detailed method for determining musical instrument fret intervals, nut offsets, and bridge/saddle offsets based on actual fretted tension. The ultimate goal being to provide an instrument with a much improved intonation profile throughout the fingerboard given a specific set of circumstances based on string properties, action height, scale length, etc. It is highly recommended that the user develop a comprehensive spreadsheet or similar computer application to facilitate this process. A software program would not only allow the user to create a tension-compensated fingerboard, nut, and bridge saddle, but also allow them to study different scenarios and variables while scrutinizing the results.

The following detailed procedure illustrates one way this can be achieved *(see fig. 3)*, however anyone skilled in the art could arrive at a similar conclusion using a variety of other mathematical or empirical means.

Procedure

1) **Select a set of strings to use and supply the following property values for each string. This data will be used for the remainder of the procedure:**

   m_{ul} = String mass per unit length, lbs./ inch
   A = Area, core wire, $in.^2$
   E = Modulus of Elasticity, core wire, psi

   If some or all of this information is unavailable then the user may have to extract this data directly from the strings through testing. See the section entitled *"String Evaluation"* for more information on this subject. Once these properties are known, start by calculating the open string tension *(T)* for each of the strings based on their open string frequencies *(see Pitch/Frequency Chart)*:

$$T = (m_{ul} * (2 * L * f)^2) / g$$

   where,
   T = Tension, lbs.

L = Scale Length, in.

f = Frequency, Hz.

g = Gravity, 386.089 in./sec^2

Next, calculate the original *"unstressed"* string length *(L_{or})* for each string using the following equation:

$$L_{or} = L / ((T / (E * A)) + 1)$$

2) **Create a set of *"rule of 18"* derived fret intervals, or equivalent, as a location reference for knowing where to press the string down at each interval. Use the formulae shown on the first few pages of this disclosure to facilitate this process.**

   Next *"graphically"* press each string down to each fret interval *(and beyond depending on touch preferred)*. Determine how much longer the strings grow or stretch beyond the basic scale length. The total fretted *(stretched)* length *(L_s)* must be found for each string pressed at each fret interval *(n)*. This can be facilitated with the help of a CAD *(computer-aided-design)* program or string geometry section within a spreadsheet software application.

If you decide to use a CAD program to facilitate this process the following procedure is one way you can accomplish this. Make sure you draw as carefully and accurately as possible since the dimensions you derive from this will be used to calculate fretted tension and ultimately compensated fret placement.

Start by drawing a side profile of the fingerboard with the "rule of 18" derived frets shown. Include a nut and saddle shown at the uncompensated location exactly the "scale length" distance apart. Knowing the 1st fret clearance dimension (action) and the

12th fret clearance dimension, draw a line between these two points above the frets, then extend the line to the nut and then to the saddle. Now you have the open string depicted at the preferred action height. If you check the actual length (at least four decimal places) of this string you will find that it is a little longer than the actual scale length. This is because it is on a slight angle due to the action. Now create a similar profile for each of the remaining strings since the action will vary slightly from string-to-string.

Next starting with the first string profile, draw a line from the saddle to the top of the 1st fret. Then draw a line from the top of the 1st fret to where your finger would be. And finally draw a line from where your finger would be to the nut. You can join the pieces together into a single "polyline" or just trim them to each other before adding up their lengths (at least four decimal places) to arrive at the ***total fretted length (L_s)*** *for the first string at the first fret. At the remaining fret locations (ex. 2 thru 20) there will probably be more than (3) string sections to add up.*

Repeat this process the same way for all fret locations and for all strings. If you have (20) frets & (6) strings you will end up with (120) values for total fretted length (L_s). I believe you can see where a spreadsheet would come in handy.

3) With the total fretted length (L_s) determined for each string and fret interval (n), the stretched or fretted tension (T_s) can be found. Calculate the fretted tension (T_{s1}, T_{s2}, T_{s3}, ...) for each string at each fret interval:

$$T_{sn} = ((L_{sn} - L_{or}) / L_{or}) * E * A$$

where,

L_{sn} = ***Length, fretted, in., (L_s at fret position "n")***

L_{or} = ***Length, original unstressed, in.***

4) **With the fretted tensions *(T_{sn})* known for each string at each fret interval, the tension-compensated fret positions can now be calculated.** ***You will be creating a complete set of fret intervals for each string based on a common scale length*****. In order to proceed we also need to know what the pitch frequencies *(f_n)* are at each of these intervals *(see Pitch/Frequency [Hz.] Chart).* The results will give you the vibrating lengths *($L_{fret\ n}$)* from the bridge saddle offset to the fret intervals.**

Calculate the fret placement *($L_{fret\ 1}$, $L_{fret\ 2}$, $L_{fret\ 3}$,…)* for each string at each fret interval as measured from the bridge saddle offset:

$$L_{fret\ n} = SQRT((g * T_{sn})/ m_{ul}) / (2 * f_n)$$

where,

g = ***Gravity, 386.089 in./sec^2***

T_{sn} = ***Tension, fretted, lbs., (T_s at interval "n")***

m_{ul} = ***String mass per unit length, lbs./in.***

f_n = ***Frequency, Hz., (f at interval "n")***

What the process should reveal to you at this point is that ***each string's fret placement varies from string-to-string*** *by a significant amount even though they share the same scale length (see fig. 4). This is to say that each string will have its own dimensionally unique set of tension compensated fret intervals given a common scale length. The reason for this is because each string has its own stiffness value and action height. These properties cause each string's fret placement to move as a set, either closer to, or farther away from the saddle offsets. It even causes the fret-to-fret spacing to vary between strings.*

These variations do increase the complexity of the problem somewhat. If we just line up the fret sets, for example let's say at the 1st fret, and average the remaining fret locations for all the strings, we end up with a set of tension-compensated fret intervals that are derived from dimensional windows at each interval that can be quite large for a set of medium acoustic guitar strings. This would produce intonation errors from various strings and fret locations that could fluctuate from approximately 4 to 15 cents (one cent = 1/100 of a semitone). As a design baseline this would not be acceptable.

In order for this method to produce an optimal intonation profile, each string's tension-compensated fret spacing must be made more dimensionally in synch with every other string's fret spacing. The total average difference between them must be reduced to just fractions of a cent to be acceptable. ***This can be accomplished by varying the scale length of each string and recalculating the results until all six strings share a dimensionally common set of fret intervals.*** *You can visualize this as working something like an accordion. When you increase the scale length the frets spread apart and when you reduce the scale length the frets squeeze closer together. When the fret spacing's are all aligned and matching between the strings you'll end up with an accurate set of tension compensated fret intervals. And by default you will also end up with the distances between the nut and first fret, which are different for each string (compensated nut), and the distances between the first fret and the saddle, which are also different for each string (compensated bridge saddle).*

The following continuation of the procedure explains how to dimensionally unify the fret intervals for all strings.

5) **With the tension-compensated fret intervals *($L_{fret\ n}$)* established for each string, first select a specific starting fret interval to be used as a *"ZERO-out" (datum-0)* location to align all string fret sets to. For example, using fret location 3 *($L_{fret\ 3}$)*, align and affix all string fret sets so they line up**

exactly at fret 3. Hold each string's fret set together at this location during the following steps.

6) **Select a second fret interval farther up the fingerboard some distance apart from the *ZERO-out* fret to be used as an *"EVEN-out" (floating)* fret location. For example, let's select fret 14 *($L_{fret\ 14}$)* for the *EVEN-out* fret location. Now calculate the *"average"* distance between fret 3 and fret 14 for all string fret sets *(see fig. 5)*.**

7) **Using the calculated average distance *(d_{avg})* between frets 3 and 14 as a reference, start varying the *scale length (L)* for each string and recalculate the entire application. This will cause *EVEN-out* fret 14 to start shifting either longer or shorter depending on the new scale length chosen. Continue varying the scale lengths until the *EVEN-out* fret for all strings are in dimensional alignment to each other. The distance between fret 3 and fret 14 for all string fret sets should equal the reference average distance *(d_{avg})* within three decimal places *(see fig. 6)*.**

   To speed this process, use the equation below to derive a *suggested scale length (L_{sug})*. Enter the result into the application and recalculate. This may have to be performed two or three times until *(d)* equals *(d_{avg})*. Also remember to input the updated variables *(L & d)* each time before recalculating.

$$L_{sug} = (L * d_{avg}) / d$$

where,

L_{sug} = ***suggested scale length, in. (single string)***

d_{avg} = ***average distance, in. (ZERO-out to EVEN-out for all strings)***

d = ***distance, in. (ZERO-out to EVEN-out for a single string)***

8) **The fret intervals for all strings should line up exactly together at frets 3 and 14. With all strings still in alignment at fret 3, you will find that the distance between fret 3 and the nut will be different for each string. This will give you the dimensional offsets for a compensated nut. The same holds true for the distances between fret 3 and the bridge. The differences here will yield the dimensional offsets for a compensated bridge saddle *(see fig. 7)*. The remaining fret locations *(1, 2, 4-13, and 15-24)* can be averaged for the different strings. Now that two fret locations are in alignment, the calculated error from averaging the remaining fret locations will be reduced to just fractions of a cent.**

Conclusion

There are a few things to be careful of when attempting this procedure. For one, there are many subtleties that can affect the outcome. They can range from fret geometry, string action, core wire properties to finger pressure. However one of the most important considerations is accurately obtaining the total fretted *(stretched)* lengths *(L_s)* for all fret intervals. Make sure you do this operation carefully. After all this is what your compensating for.

In general no quantity or property, regardless of its size, can be ignored because deriving a useable result is a *cumulative* effort. Forces and deflections on this scale are part of the *micro* world, not the *macro.* Get used to using decimal places.

String Evaluation

To perform accurate fret interval calculations it is very important to have precise string property data. However, it is most unfortunate that many, if not all, musical instrument string manufacturers are reluctant to share this information, especially their core wire data. It appears that if we want meaningful data to work with, then we will have to extract this information from the strings ourselves. To be more specific we need to know the following string properties:

m_{ul} = *String mass per unit length, lbs. per linear inch*
A = *Area, core wire, in.*^{2}
E = *Modulus of Elasticity, core wire, psi*

In the most critical formulae the product of the two *(E * A)* is acceptable in lieu of individual values. This data can be extracted directly from the strings using a test fixture. The fixture will have a known and reasonably accurate scale length *(L)* spanning two supports, one of which will be moveable for stretching the sample string a known distance. This can be accomplished with a sliding bed or rack with the moveable support mounted to it. The movement can be controlled by a *"draw-screw"* such as a ½"-20 bolt or equivalent and monitored by a dial indicator for accuracy. The string will be put under tension *(T)* with accurately measured weights or with a tuner at one end and a force gauge at the other. Once stabilized, the string will be lightly plucked or excited in some consistent manner *(for frequency stability)* and it's frequency will be measured directly with a *"frequency counter"* such as one that may be included as part of a digital strobe tuner or related software. The frequency's unit of measure will need to be in *"hertz" (Hz)*. Once the frequency *(f)* and tension *(T)* are known, the mass per unit length (m_{ul}) in *"lbs. per linear inch"* can be derived using the following equation:

$$m_{ul} = (g * T) / (2 * L * f)^2$$

where,

g = *Gravity, 386.089 in./sec*^{2}

T = *Tension, lbs.*

L = *Scale Length, in.*

f = *Frequency, Hz.*

Next, the string will be locked or clamped down at the weight or force gauge side to prevent slippage. The string's frequency should be checked again to make sure it's reading hasn't changed or adjusted if it has. Now the string can be stretched using the draw-screw to shift the moveable support a known distance *(as confirmed by a dial indicator).* The amount of movement should probably be in the 0.005-0.010 range lengthening or stretching the string. At this point it's increased frequency will be measured again. With the extended total length (L_s) known and the higher frequency (f_s) measured, the new higher tension (T_s) can now be calculated using the following equation:

$$T_s = (m_{ul} * (2 * L_s * f_s)^\wedge 2) / g$$

where,

L_s = *Length, stretched, in.*

f_s = *Frequency, stretched, Hz.*

Calculate the length of original stretch (ΔL_{or}) from no-load to full tension (T) using the following equation:

$$\Delta L_{or} = T / ((1/\Delta L_s) * \Delta T_s)$$

where,

ΔL_s = $L_s - L$, *in.*

ΔT_s = $T_s - T$, *lbs.*

Calculate the original no-load string length *(L_{or})* before tension was applied using the equation:

$$L_{or} = L - \Delta L_{or}$$

Now calculate the modulus of *Elasticity x Area (E * A)*:

$$(E*A) = L_{or} * (1/\Delta L_s) * \Delta T_s$$

where,

E = *Modulus of Elasticity, core wire, psi*

A = *Area, core wire, in.*^{2}

E_2 - Guitar 6	82.407	A_3	220.000	D_5	587.330
F_2	87.307	$A^{\#}_3$	233.082	$D^{\#}_5$	622.254
$F^{\#}_2$	92.499	B_3 - Guitar 2	246.942	E_5	659.255
G_2	97.999	**C_4**	**261.626**	F_5	698.456
$G^{\#}_2$	103.826	$C^{\#}_4$	277.183	$F^{\#}_5$	739.989
A_2 - Guitar 5	110.000	D_4	293.665	G_5	783.991
$A^{\#}_2$	116.541	$D^{\#}_4$	311.127	$G^{\#}_5$	830.609
B_2	123.471	E_4 - Guitar 1	329.628	A_5	880.000
C_3	130.813	F_4	349.228	$A^{\#}_5$	932.328
$C^{\#}_3$	138.591	$F^{\#}_4$	369.994	B_5	987.767
D_3 - Guitar 4	146.832	G_4	391.995	C_6	1046.502
$D^{\#}_3$	155.563	$G^{\#}_4$	415.305	$C^{\#}_6$	1108.731
E_3	164.814	A_4	440.000	D_6	1174.659
F_3	174.614	$A^{\#}_4$	466.164	$D^{\#}_6$	1244.508
$F^{\#}_3$	184.997	B_4	493.883	E_6	1318.510
G_3 - Guitar 3	195.998	C_5	523.251		
$G^{\#}_3$	207.652	$C^{\#}_5$	554.365		

Pitch / Frequency (*Hz*) Chart